Cavendish–HEP–94/7
hep-ph/9408222

Estimation of Power Corrections to Hadronic Event Shapes*

B.R. Webber
Cavendish Laboratory, University of Cambridge,
Madingley Road, Cambridge CB3 0HE, U.K.

Abstract

Power corrections to hadronic event shapes are estimated using a recently suggested relationship between perturbative and non-perturbative effects in QCD. The infrared cutoff dependence of perturbative calculations is related to non-perturbative contributions with the same dependence on the energy scale Q. Corrections proportional to $1/Q$ are predicted, in agreement with experiment. An empirical proportionality between the magnitudes of perturbative and non-perturbative coefficients is noted.

Cavendish–HEP–94/7
July 1994

*Research supported in part by the U.K. Particle Physics and Astronomy Research Council and by the EC Programme "Human Capital and Mobility", Network "Physics at High Energy Colliders", contract CHRX-CT93-0537 (DG 12 COMA).

Event shapes in the process $e^+e^- \to$ hadrons have been widely used to test QCD and to determine its coupling constant α_S. Predictions of infrared safe quantities from perturbation theory, either in next-to-leading order or with enhanced terms summed to all orders, generally provide a good description of the data, provided they are subjected to "hadronization corrections" obtained from non-perturbative models. For most quantities these corrections appear empirically to be proportional to $1/Q$ where Q is the centre-of-mass energy. This is in contrast to the total cross section, which for massless quarks has a leading power correction of order $1/Q^4$. The smallness of non-perturbative effects in the total cross section and related quantities, such as the hadronic widths of the Z^0 boson and the τ lepton, has led to a preference for these quantities as a means of determining α_S, even though event shapes have a stronger perturbative dependence on α_S.

In the case of the total cross section, we also have some understanding of the leading power correction. It is believed to arise from the vacuum expectation value of the gluon condensate, $\langle \alpha_S G^2 \rangle$, which is the relevant gauge-invariant operator of lowest dimension, giving a correction proportional to $\langle \alpha_S G^2 \rangle / Q^4$. For event shapes, we do not even know why the corrections should be of order $1/Q$: there are no operators of dimension one to which they could be related.

Another way of discussing power corrections is in terms of renormalons [1]. These are singularities of the Borel transform of the all-orders perturbative expression for a quantity, generated by a factorial growth of the perturbation series at high orders.† Such growth in the soft region is thought to give rise to an infrared renormalon at the position $8\pi/\beta_0$ ($\beta_0 = 11 - \frac{2}{3}N_f$) in the Borel plane, corresponding to a power correction proportional to

$$\exp\left[-\frac{8\pi}{\beta_0 \alpha_S(Q^2)}\right] \sim \frac{\Lambda^4}{Q^4} \tag{1}$$

where Λ represents the QCD scale. The existence of the renormalon is supposed to indicate that the full QCD prediction would exhibit the same power correction. In this language, the appearance of $1/Q$ corrections to event shapes would be associated with a new infrared renormalon at $2\pi/\beta_0$ in the Borel transforms of these quantities.

†For a recent review, see Ref. [2]

In the present paper, I apply an idea due to Bigi, Shifman, Uraltsev and Vainshtein [3], who argue that there is a simple correspondence between renormalon positions and the power corrections to fixed-order perturbative predictions evaluated with an infrared cutoff. In $e^+e^- \to$ hadrons, to first order in α_S all diagrams are QED-like and a suitable cutoff can be imposed by introducing a small mass μ in the denominator of the gluon propagator.‡ One finds that the first-order perturbative total cross section with such a cutoff, normalized to the Born value, is

$$R_p = 1 + \alpha_S/\pi - D\,\alpha_S\,\mu^4/Q^4 + \ldots \tag{2}$$

where D is a constant and the ellipsis represents non-leading power corrections. The dependence on μ must cancel between this expression and the soft contribution, which builds the renormalon. Thus the leading renormalon occurs at $8\pi/\beta_0$, as described above, and we expect a non-perturbative contribution of the form

$$R_{np} = [C\,\Lambda^4 + D\,\alpha_S(\mu)\,\mu^4]/Q^4 \tag{3}$$

where C is a constant. The μ-dependence appears as an arbitrariness in the part of the correction that we attribute to the renormalon and the part that is generated in fixed order.

In the case of event shapes, however, one finds that the introduction of a gluon mass in the way outlined above leads to corrections of order $\alpha_S\mu/Q$. That is, for a generic (infrared safe) event shape S we find in first order

$$S_p = A_S\,\alpha_S - D_S\,\alpha_S\,\mu/Q + \ldots \tag{4}$$

instead of a relation of the form (2). The coefficients D_S are easily computed; to this order, they arise entirely from the reduction of phase space for real gluon emission. Their values, together with those of the leading coefficients A_S, are listed for some representative quantities in Table 1. Here T is the thrust [4], C is the C-parameter [5], and σ_L is the longitudinal cross section [6].

‡Recall that in QED a photon mass can be introduced in this way without violating the Ward identities associated with current conservation.

S	A_S	D_S	$C_S\,\Lambda$
$\langle 1-T\rangle$	0.335	$\frac{16}{3\pi}=1.7$	~ 1.0 GeV
$\langle C\rangle$	1.375	8	~ 5.0 GeV
σ_L	$1/\pi$	4/3	~ 0.8 GeV

Table 1: Coefficients of terms in Eqs. (4) and (5).

From Eq. (4) we expect that a new infrared renormalon at $2\pi/\beta_0$ is present in event shapes, leading to a non-perturbative contribution

$$S_{np} = [C_S\,\Lambda + D_S\,\alpha_S(\mu)\,\mu]/Q\ , \tag{5}$$

whose dependence on the arbitrary cutoff μ cancels against that of the perturbative part, leaving a cutoff-independent power correction $C_S\,\Lambda/Q$. The observed value of this correction, inferred [7] from experimental data, is shown for each quantity in Table 1.

It is remarkable that the observed $1/Q$ corrections are, within the uncertainties, proportional to the perturbative coefficients D_S, suggesting that Eq. (5) takes the general form

$$S_{np} = C_S\frac{\Lambda}{Q}\left[1 + d\,\alpha_S(\mu)\frac{\mu}{\Lambda}\right] \tag{6}$$

where d is a constant, roughly equal to 0.5 if we take $\Lambda \simeq 0.3$ GeV. Since $\alpha_S(\mu)\,\mu/\Lambda$ has a minimum value (at $\mu = e\Lambda$) of $2\pi e/\beta_0 \simeq 1.9$, the cutoff-dependent contribution is comparable to, or greater than, the full $1/Q$ correction. This makes the applicability of the method marginal: it would be preferable if there existed a region of μ in which the cutoff dependence was small compared with both the perturbative and non-perturbative contributions, as discussed in Ref. [3].

It would obviously be of interest to apply the above approach to a wide variety of quantities, and to try to construct event shapes from which $1/Q$ corrections are absent. From Table 1 we see that the combination $\langle T + 2C/3\pi\rangle$

might be of this type. It would be desirable to extend the treatment to higher orders in perturbation theory, but it is difficult to see how this can be done in a gauge-invariant way, unless the dimensional regularization method can be adapted to the purpose.

I am grateful to Steven Bass and Mike Seymour for comments.

References

1. A.V. Manohar and M.B. Wise, Univ. of California at San Diego preprint UCSD/PTH 94-11.

2. A.H. Mueller, in *QCD 20 Years Later*, vol. 1 (World Scientific, Singapore, 1993).

3. I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Univ. of Minnesota preprint TPI-MINN-94/4-T (CERN-TH.7171/94).

4. E. Farhi, Phys. Rev. Lett. 39 (1977) 1587.

5. R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. B178 (1981) 421.

6. P. Nason and B.R. Webber, Nucl. Phys. B421 (1994) 473.

7. B.R. Webber, to appear in *Proc. 27th Int. Conf. on High Energy Physics*, Glasgow, 1994; see also *Proc. Workshop on New Techniques for Calculating Higher-Order QCD Corrections* (ETH, Zurich, 1992).